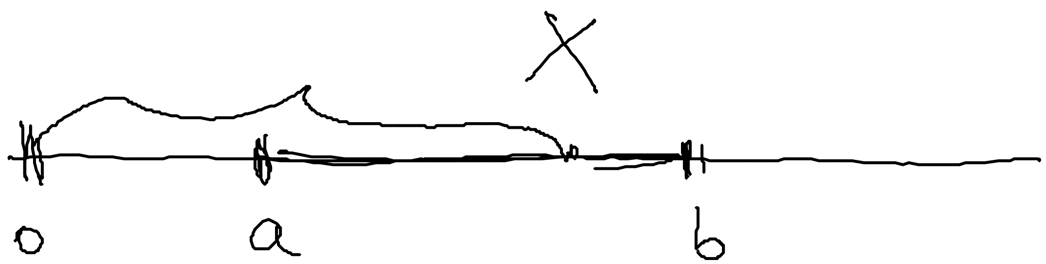
$$I \longmapsto P(X \in I)$$

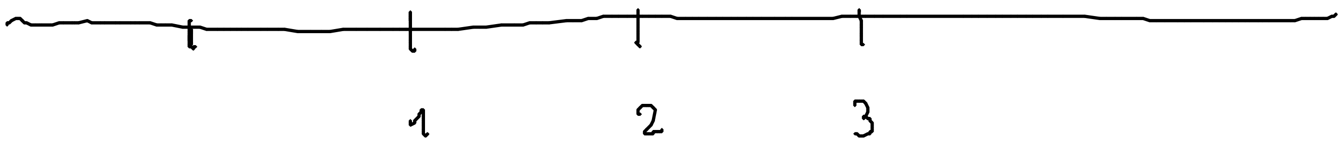
Legge uniforme sull'intervallo $[a,b]$



$$P(X = x) = \alpha$$

Vogliamo "scegliere un punto a caso in $[a,b]$"

X = ascissa del punto scelto

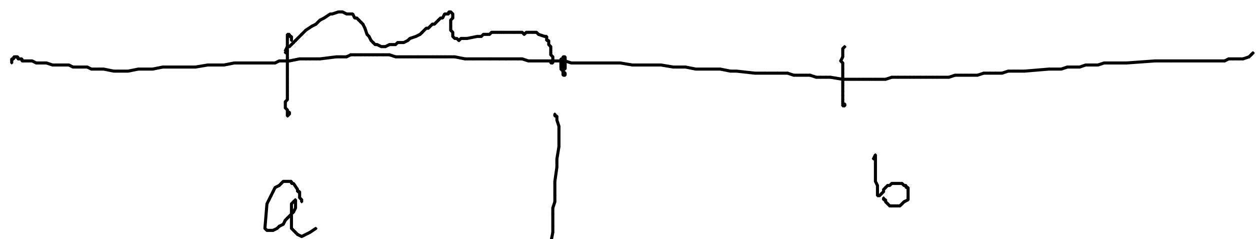


Scelgo un punto "a caso" nell'intervallo di interi $[1, 3]$

$$X = \begin{cases} 1 & 1/3 \\ 2 & 1/3 \\ 3 & 1/3 \end{cases}$$

$$P(X = x)$$

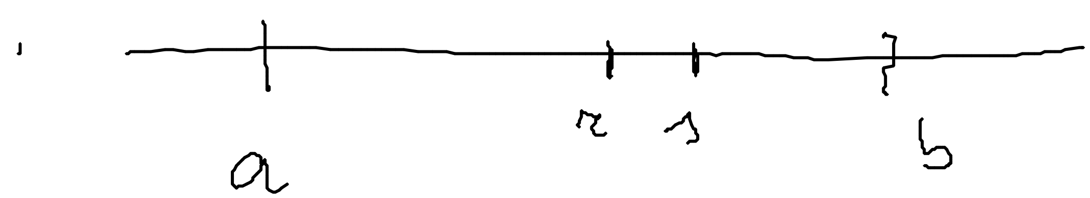
$$P(X = k) = \begin{cases} \frac{1}{3} & k = 1, 2, 3 \\ 0 & \text{———} \end{cases}$$

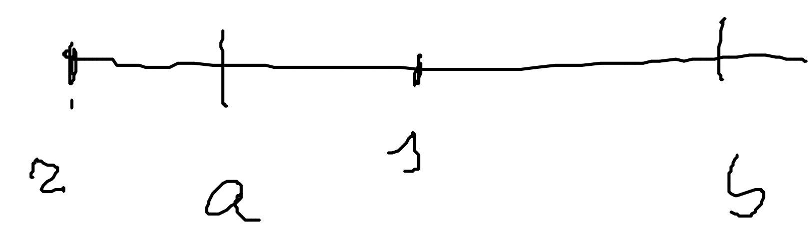


$$m = \frac{a+b}{2}$$

$$P(X \in [a, m]) = \frac{1}{2}$$

$$P(X \in [r, s]) = \frac{s-r}{b-a} \qquad \frac{s-a}{b-a}$$





$$F(t) = P(X \le t) = \begin{cases} 0 & t < a \\ \dfrac{t-a}{b-a} & a \le t \le b \\ 1 & t > b \end{cases}$$

$$P(X \in [r,s]) = \frac{\ell([r,s] \cap [a,b])}{b-a}$$

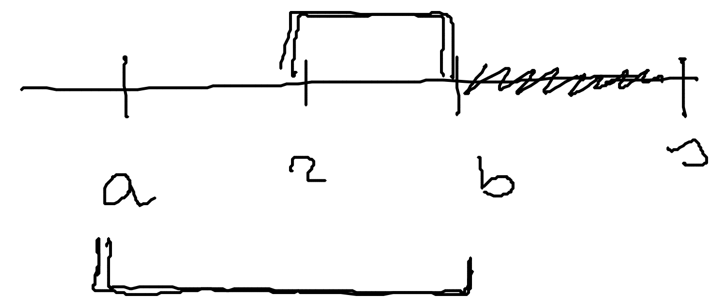
$$P(X \in (r,s]) = P(r < X \le s) =$$

$$= P(X \le s) - P(X \le r) =$$

$$= F(s) - F(r) = \frac{s-a}{b-a} - \frac{r-a}{b-a} =$$

$$= \frac{s-r}{b-a}$$

$$P(X \in (r, s]) =$$



$$= P(X \le s) - P(X \le r) =$$

$$= 1 - \frac{r-a}{b-a} = \frac{b-r}{b-a}$$

$$P(X \in [r, s]) = P(r \le X \le s) =$$

$$= P(X \le s) - P(X < r)$$

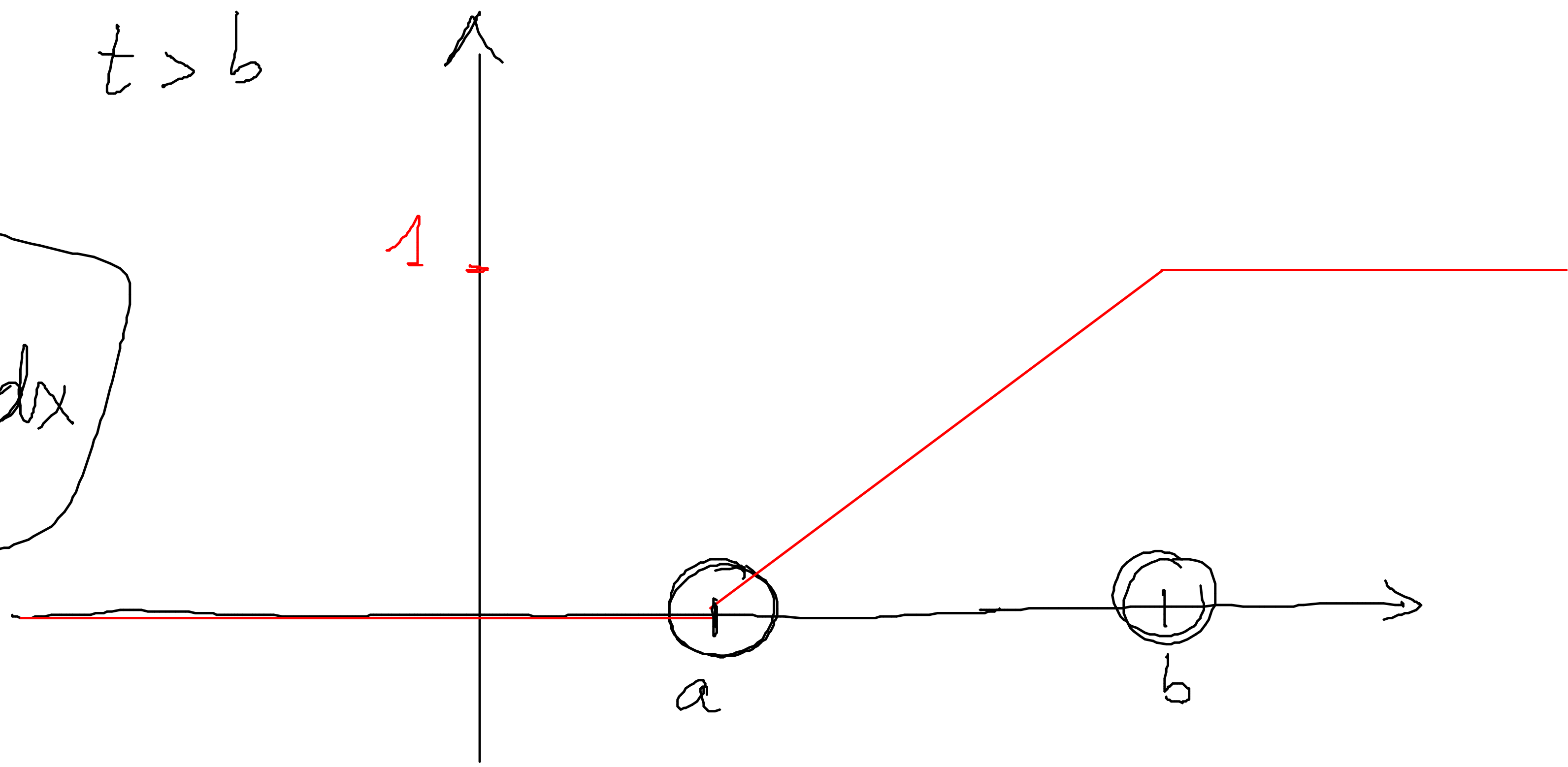
$$\boxed{P(X = r) = 0}$$

$$F(t) = \begin{cases} 0 & t < a \\ \dfrac{t-a}{b-a} & a \le t \le b \\ 1 & t > b \end{cases}$$

$x \mapsto P(X = x)$

$$P(X \in A) = \int_A f(x)\,dx$$

densité



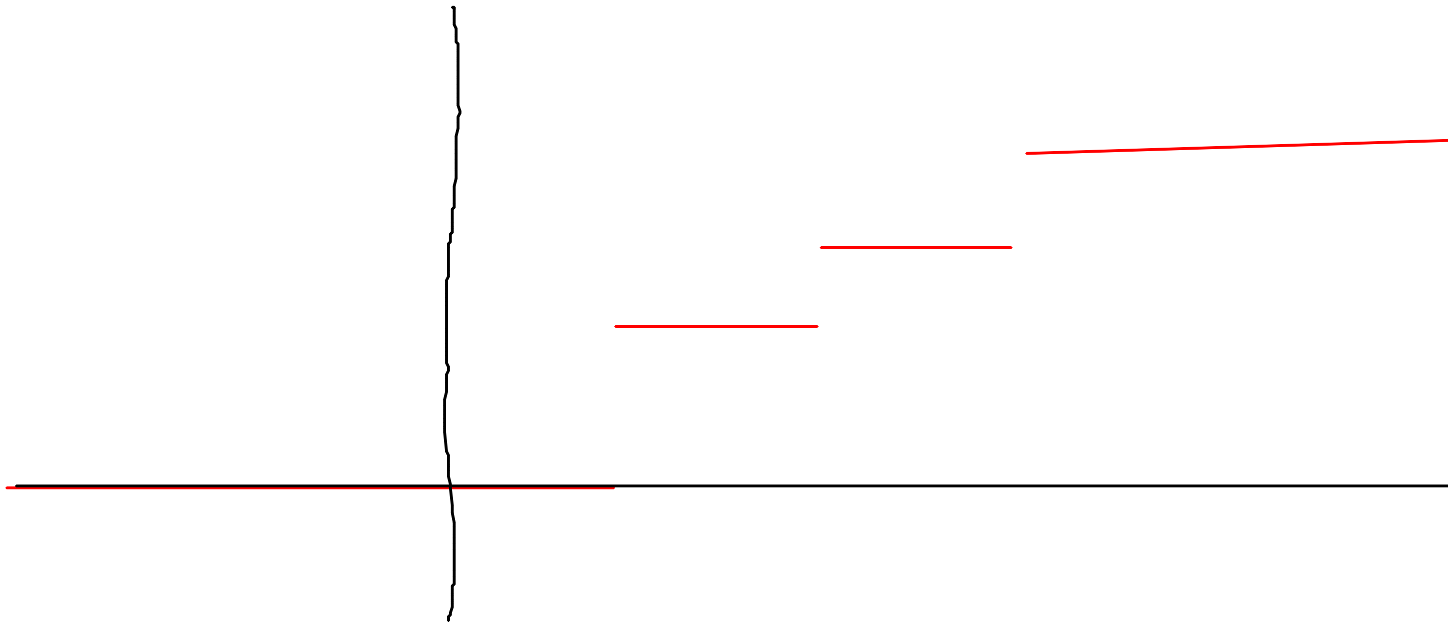
$$\underline{F(t)} = \int_{-\infty}^{t} f(x)\,dx =$$

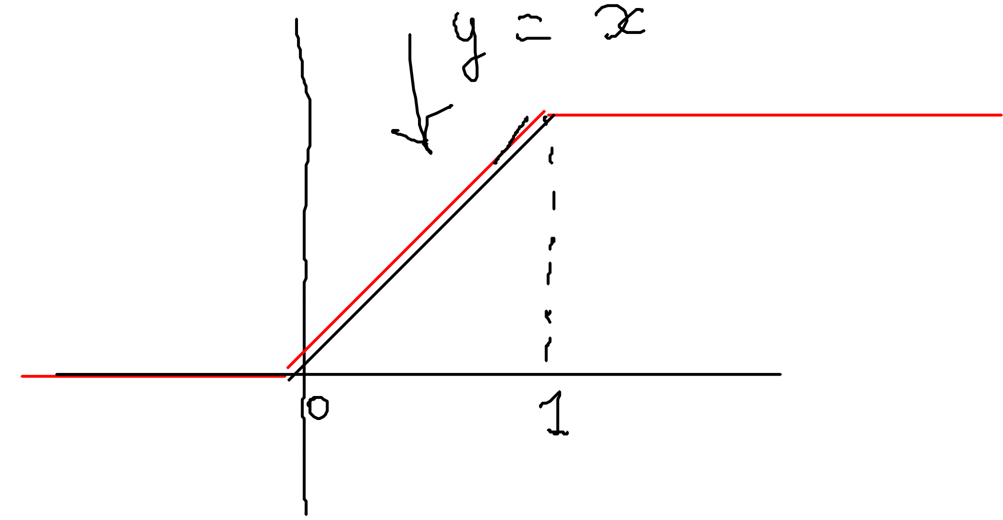
$$= \boxed{\int_{-\infty}^{a} f(x)\,dx} + \int_{a}^{t} f(x)\,dx$$

$$F'(t) = \frac{d}{dt} \boxed{\int_{a}^{t} f(x)\,dx}$$

$$F(t) = \begin{cases} 0 & t < a \\ \dfrac{t-a}{b-a} & a \leq t \leq b \\ 1 & t > b \end{cases}$$

$$g(t) = \begin{cases} 0 & t \leq a \\ \dfrac{1}{b-a} & a < t < b \\ 0 & t \geq b \end{cases}$$





$$X \sim \mathcal{U}([a,b])$$

$$E[X] = \,? \qquad \frac{a+b}{2}$$

$$E[X] = \int_{-\infty}^{+\infty} x\, f(x)\, dx = \cancel{\int_{-\infty}^{+\infty} x \cdot \frac{1}{b-a}\, dx}$$

$$= \cancel{\int_{-\infty}^{a} x \cdot 0\, dx} + \int_{a}^{b} x\, \frac{1}{b-a}\, dx + \cancel{\int_{b}^{+\infty} x \cdot 0\, dx} =$$

$$= \frac{1}{b-a} \int_{a}^{b} x\, dx = \frac{1}{b-a} \left[\frac{x^2}{2} \right]_{a}^{b} = \frac{b^2 - a^2}{2} \cdot \frac{1}{b-a}$$

$$= \frac{a+b}{2}$$

$$\operatorname{Var} X = E[X^2] - E^2[X] =$$

$$\int_{-\infty}^{+\infty} x^2 f(x)\,dx \quad - \quad \left(\frac{a+b}{2}\right)^2$$

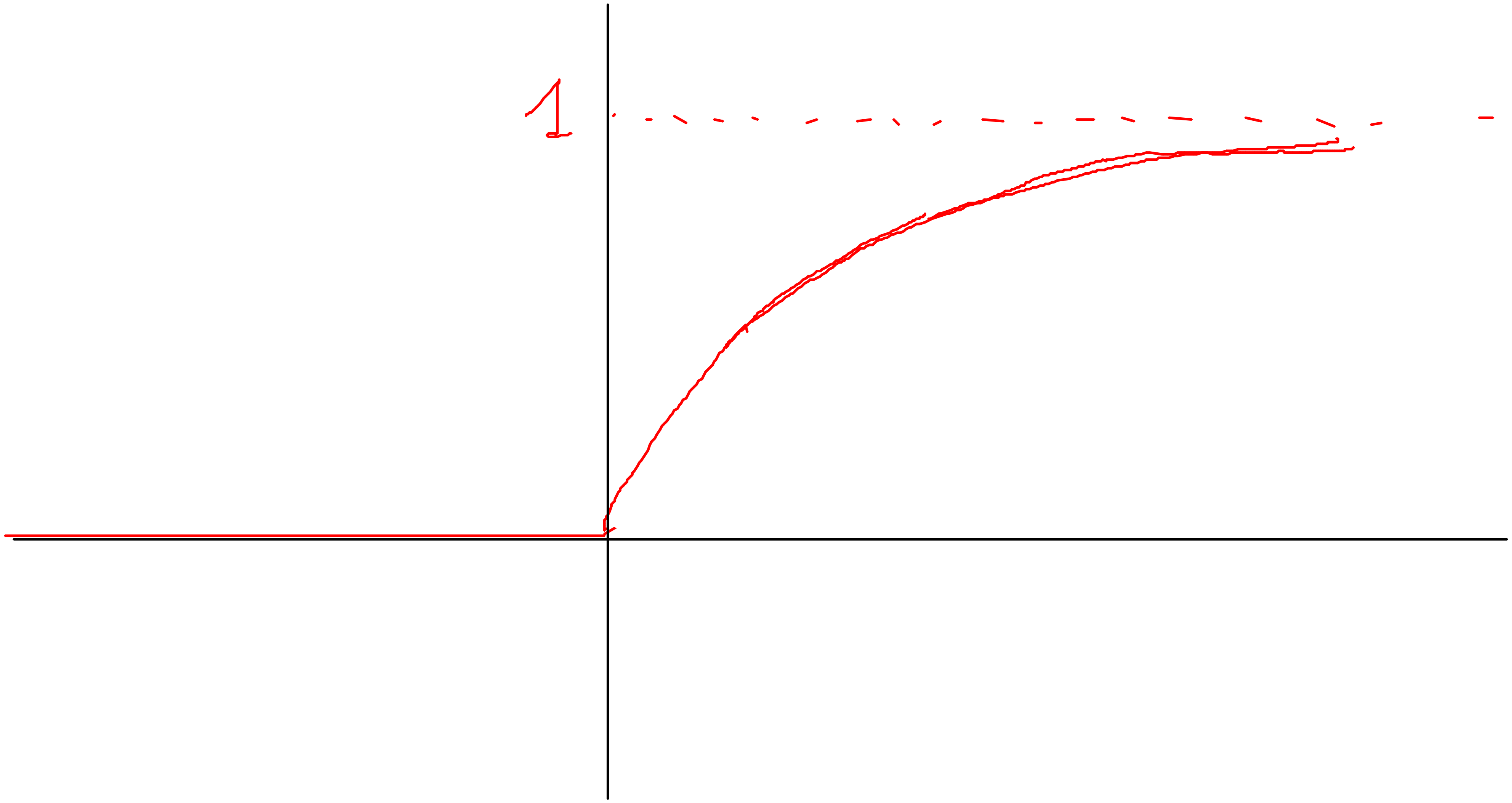
$$\| $$

$$\int_a^b x^2 \frac{1}{b-a}\,dx = \ldots$$

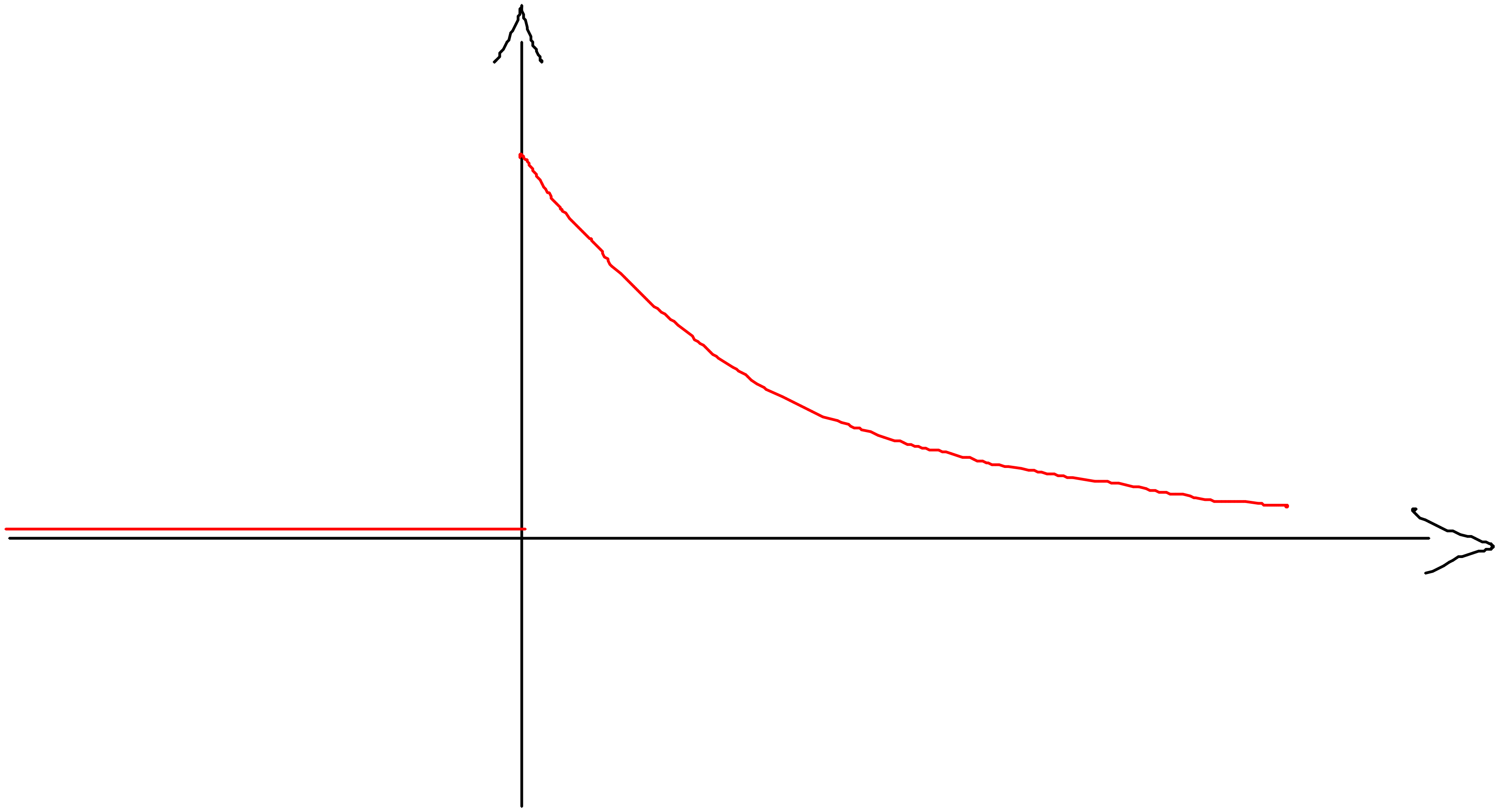
Legge esponenziale (di parametro $\lambda > 0$)

$$X \sim \mathcal{E}(\lambda)$$

$$F(t) = \begin{cases} 0 & t < 0 \\ \underline{1 - e^{-\lambda t}} & t \geq 0 \end{cases}$$



$$f(x) = \begin{cases} 0 & t < 0 \\ \lambda e^{-\lambda t} & t \geq 0 \end{cases}$$



X = durata di una lampadina

$X \sim \mathcal{E}(\lambda)$

Proprietà di mancanza di memoria
assenza di usura

$\forall\, t, s > 0$

$$P(X > t + s \mid X > t) = P(X > s)$$

Verifica

$P($

$$P(X > t+s \mid X > t) =$$

$$= \frac{P(\underbrace{X > t+s}, \underbrace{X > t})}{P(X > t)} = \frac{P(X > t+s)}{P(X > t)}$$

$x > 0$

$$\boxed{P(X > x) = 1 - P(X \le x) = 1 - F(x) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x}}$$

$$\cdot = \frac{e^{-\lambda(t+s)}}{e^{-\lambda t}} = e^{-\lambda s} = \underline{P(X>1)}$$

$$t \mapsto P(X \le t)$$

$t \longmapsto P(X > t)$ funzione di sopravvivenza (survival function)

$$\boxed{X \sim \mathcal{E}(\lambda)}$$

Osservazione.

$$X \sim \underline{f(x)} = \begin{cases} \underline{h(x)} & \underline{x \in I} \\ 0 & x \notin I \end{cases}$$

$\Longrightarrow P(X \in I) = 1$. Dimostrazione

$$P(X \in I) = \int_I f(x)\,dx = \int_I h(x)\,dx + \int_{I^c} 0\,dx = \int_{\mathbb{R}} f(x)\,dx = 1$$

$$X \sim \mathcal{E}(\lambda)$$

$$E[|X|] = E[X] = \int_{-\infty}^{+\infty} x \, f(x) \, dx =$$

$$= \cancel{\int_{-\infty}^{0} x \cdot 0 \, dx} + \int_{0}^{+\infty} x \cdot \lambda e^{-\lambda x} \, dx =$$

$$E[X] = \lambda \int_0^{+\infty} x \cdot \underline{e^{-\lambda x}}\, dx =$$

$$\lim_{x \to +\infty} x\, e^{-\lambda x} = 0$$

$$= \lambda \left\{ \cancel{\frac{e^{-\lambda x}}{-\lambda}\, x \Big|_0^{+\infty}} - \int_0^{+\infty} \frac{e^{-\lambda x}}{-\lambda}\, dx \right\} =$$

$$= \cancel{\lambda} \left\{ \frac{1}{\cancel{\lambda}} \int_0^{\infty} e^{-\lambda x}\, dx \right\} = \frac{e^{-\lambda x}}{-\lambda} \Big|_0^{+\infty} = \frac{1}{\lambda}$$

$$E[X^2] =$$

$$\int_0^{\infty} x^2 e^{-\lambda x}\, dx$$

Leggi di Weibull

$$f(x) = \begin{cases} \lambda \alpha x^{\alpha-1} e^{-\lambda x^{\alpha}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$\alpha > 0$

$\lambda > 0$

(i) non negativa

(ii) $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\varphi : \underline{t} \mapsto \underline{P(X > t+s \mid X \underline{\geq} t)} \overset{\substack{\text{esponenziale} \\ \downarrow}}{=} P(X > s)$$

Cerchiamo condizioni su α e λ

in modo che φ risulti decrescente

$$P(X > t+s \mid X > t) = \frac{P(X > t+s)}{P(X>t)} = \frac{e^{-\lambda (t+s)^{\alpha}}}{e^{-\lambda t^{\alpha}}}$$

$$x > 0$$

$$P(X > y) = \int_{y}^{+\infty} f(x)\, dx =$$

$$= \int_{y}^{+\infty} \underbrace{\lambda \alpha x^{\alpha-1}} e^{-\lambda x^{\alpha}} \underbrace{dx} = \int_{y^{\alpha}}^{+\infty} \underline{\lambda e^{-\lambda z}}\, dz =$$

$$x^{\alpha} = z$$

$$Z \sim \mathcal{E}(\lambda) = P(Z > y^{\alpha}) = e^{-\lambda y^{\alpha}}$$

$$\varphi : t \longmapsto \frac{e^{-\lambda(t+s)^{\alpha}}}{e^{-\lambda t^{\alpha}}} = e^{-\lambda\left((t+s)^{\alpha} - t^{\alpha}\right)}$$

$\alpha < 1$

φ decrescente

$$h : t \longmapsto (t+s)^{\alpha} - t^{\alpha} \quad \text{crescente}$$

$$h'(t) = \alpha(t+s)^{\alpha-1} - \alpha t^{\alpha-1} \geq 0$$

$$(t+s)^{\alpha-1} \geq t^{\alpha-1} \longrightarrow \alpha \geq 1$$

$$E[X] = \int_0^{+\infty} x\,\alpha\lambda x^{\alpha-1} e^{-\lambda x^{\alpha}}\,dx = \lambda \int_0^{\infty} y^{1/\alpha} e^{-\lambda y}\,dy$$

$$= \int_0^{\infty} \alpha\lambda x^{\alpha} e^{-\lambda x^{\alpha}}\,dx$$

$x^{\alpha} = y \qquad \alpha x^{\alpha-1}\,dx = dy$

$x = y^{1/\alpha}$

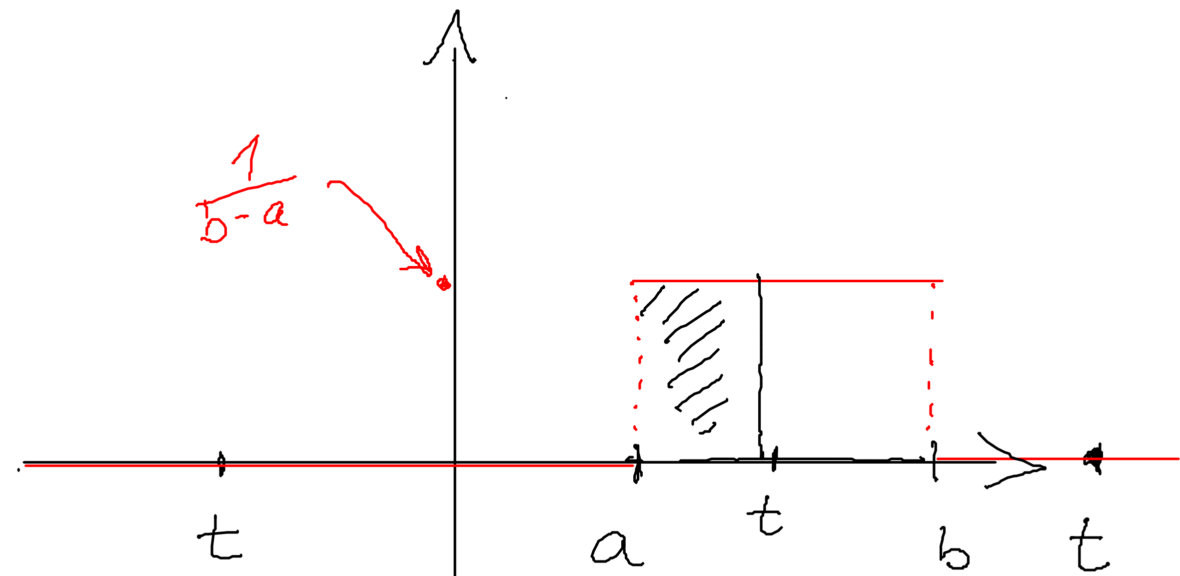
$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{0}^{+\infty} \lambda \alpha x^{\alpha-1} e^{-\lambda x^{\alpha}} dx =$$

$$x^{\alpha} = y \qquad dy = \alpha x^{\alpha-1} dx$$

$$= \int_{0}^{+\infty} \lambda e^{-\lambda y} dy = 1 -$$

$$F(t) = P(X \le t) \overset{?}{=} \int_{-\infty}^{t} g(x)\,dx \qquad \forall t$$

$$\int_{-\infty}^{t} g(x)\,dx = 0 \qquad \text{for } t < a$$



$a \le t \le b$

$$\int_{-\infty}^{t} g(x)\,dx = \int_{-\infty}^{a} + \int_{a}^{t} = 0 + \frac{t-a}{b-a}$$

$$\int_{-\infty}^{t} = \cancel{\int_{-\infty}^{a}} + \int_{a}^{b} + \cancel{\int_{b}^{t}}$$

$$F(t) = P(X \le t) = P(X \in (-\infty, t])$$

$$= \underbrace{\int_{-\infty}^{t} f(x)\, dx}$$

$$F(t) = \int_{-\infty}^{t} f(x)\, dx$$

$$F(t) = \int_a^t f(x)\,dx$$

F = funzione integrale di f

Teorema: Se f è continua, allora F è derivabile

e $F'(x) = f(x)$